**Lecture 1 - Vectors**

Dot product 𝐮 ∙ 𝐯 = ||𝐮|| ||𝐯|| 𝑐𝑜𝑠 (𝜃), 𝜃 = arccos (𝐮 ∙ 𝐯 /||𝐮|| ||𝐯||), If two vectors are orthogonal: 𝜃 = 90°, i.e., cos(𝜃) = 0, then 𝐮 ∙ 𝐯 = 0

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Diagram

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F1 = 2/((1/Precision)+(1/Recall)=2 x ((Precsion \* Recall) / (Precision + Recall)) = TP / (TP +((FN+FP)/2))

The ROC curve compares True Positive Rate (Recall) vs False Positive Rate. A perfect classifier will ha a ROC Area Under the Curve (AUC) of 1 a Random Classifier will have AUC of 0.5. Type 1 is False Positive; Type II is False Negative.

**Lecture 2 - Matrices**

Associative: (AB)C = A(BC), m(BC) = (mB)C = B(mC) = (BC)m

Distributive: (A+B)C=AC+BC, A(C+D) = AC+AD, m(B+C)=mB+mC

For A and mxn matrix ImA = Ain = A; Inverse: A \* A-1 = I; Transpose: B = At where Bij = Aji; Properties: AA-1=I=A-1A; (AB)-1=B-1A-1; (A+B) -1 != A-1 + B-1; (AT)T = A; (AB) T=B TA T; (A+B) T=A T +B T

Elementary Transformations 1. Exchange of two equations (rows in the matrix representing the system of equations) 2. Multiplication of an equation (row) with a constant λ ∈ R\{0} 3. Addition of two equations (rows)

Particular Solutions 1. Use elementary transformations to put in Row-Echelon form 2. Set non-pivot column coefficients to 0.

General Solutions 1. Use elementary transformations to put in reduced row-echelon form. AKA Gaussian Elimination 2. Express nonpivot columns as linear combinations of pivot columns

Row Echelon Form: All rows with only zeros are on the bottom; Pivots are always to the right of pivots above; Reduced Row Echelon Form: Row Echelon Form, Every Pivot is 1, The pivot is the only non-zero entry in its column.

To calculate the inverse of A, form [A | I] transform to [I | X], X is the inverse of A

Vector spaces is closed under vector addition and multiplication by a scaler.

**Lecture 3 – Linear Equations**

Determinants – AD – CB, 3x3 axDet(ef/hi) - bxDet(df/gi) + cDet(de/gh)

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A collection of vectors 𝐯1, 𝐯2, … , 𝐯𝑘 are linearly dependent if there exist coefficients 𝑎1, 𝑎2, … , 𝑎𝑘 not all equal to zero, so that the sum of aivi from I to k = 0. If there is no linear dependence the vectors are linearly independent

For an 𝑛 × 𝑚 matrix, the rank of the matrix is the largest number of linearly independent columns

If det 𝐴 = 0 (i.e., rank 𝐴 < 𝑛), then the inverse does not exist

If the inverse of a matrix is equal to its transpose, the matrix is said to be orthogonal matrix – A-1 = AT

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**Lecture 4 – Optimization**

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Gradient descent:xi+1 = xi – lambdai((delta f)(xi))T

X0 – initial guess, x0+1 – next step, lambdai = step size

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Convex Optimization, a set is convex if a line between any two points are contained within that set. Functions are convex if the region above the function is a convex set.

Partial f wrt x = lambda \* partial g wrt x

Partial f wrt y = lambda \* partial g wrt y

g(x, y) = 0

Solve the equations